

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1

Branch: Diploma (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) The distance between the points (1, 3) and (0, -4) is _____.
(A) 50 (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) None of these
- b) If (3, 8), (4, 2) and (-1, 5) are the vertices of a triangle, then the co ordinates of its centroid is _____.
(A) (2, -5) (B) (-2, 5) (C) (2, 5) (D) None of these
- c) Slope of the line $2x - 3y + 4 = 0$ is _____.
(A) $-2/3$ (B) $-3/2$ (C) $3/2$ (D) $2/3$
- d) Radius of the circle $x^2 + y^2 = 7$ is _____.
(A) 7 (B) $\sqrt{7}$ (C) $\frac{7}{2}$ (D) None of these
- e) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x =$ _____.
(A) e^2 (B) e (C) $e^{1/2}$ (D) None of these
- f) $\lim_{x \rightarrow \infty} x \left[\sqrt[3]{7} - 1 \right] =$ _____.
(A) 0 (B) e^7 (C) $\log_e 7$ (D) 1
- g) $\lim_{x \rightarrow 1} \frac{x^2 + 3x + 4}{x + 2} =$ _____.
(A) $8/3$ (B) $3/8$ (C) 2 (D) 3
- h) $\frac{d(a^x)}{dx} =$ _____.
(A) $a^x \log_e a$ (B) $\log_e a$ (C) $x^a \log_e a$ (D) $a^x \log_e x$
- i) $\frac{d(x^2 + 2x + 7)}{dx} =$ _____.
(A) $2x$ (B) $2x + 1$ (C) $2x + 2$ (D) None of these



- j) $\frac{d(\log \cos x)}{dx} = \underline{\hspace{2cm}}$
 (A) $\cot x$ (B) $-\tan x$ (C) $\operatorname{cosec} x$ (D) $\sec x$
- k) $\frac{d(\tan^{-1} x + \cot^{-1} x)}{dx} = \underline{\hspace{2cm}}$
 (A) -1 (B) 0 (C) 1 (D) None of these
- l) $\int \sec^2 x \, dx = \underline{\hspace{2cm}}$
 (A) $\tan x + c$ (B) $\cot x + c$ (C) $\sec x \tan x + c$ (D) $-\operatorname{cosec} x \cot x + c$
- m) $\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \underline{\hspace{2cm}}$
 (A) $\log \left| x + \sqrt{x^2 + a^2} \right| + c$ (B) $\log \left| x + \sqrt{x^2 - a^2} \right| + c$ (C) $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$
 (D) $\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$
- n) $\int_0^1 \frac{2}{1+x^2} \, dx = \underline{\hspace{2cm}}$
 (A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) In which ratio Y – axis divides line segment joining points (1, 2) and (2, 1)? Find co ordinates of division point. (5)
- b) Find the equation of straight line passing through (3, 4) and parallel to line $\frac{x}{2} + \frac{y}{2} = 1$. (5)
- c) If $f(x) = \frac{1-x}{1+x}$ then prove that $f(x) + f\left(\frac{1}{x}\right) = 0$. (4)

Q-3 Attempt all questions (14)

- a) Prove that $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \frac{\sqrt{35}}{5}$. (5)
- b) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\operatorname{cosec} \theta - \cot \theta}{\theta}$ (5)
- c) A(-1, 3), B(-1, x) and C(4, 3) are vertices of a triangle. If $m\angle B = 90^\circ$ then find value of x. (4)

Q-4 Attempt all questions (14)

- a) Find centre and radius of the circle $36x^2 + 36y^2 + 24x - 36y - 23 = 0$. (5)
- b) Find derivative of $f(x) = e^x$ using definition. (5)
- c) Find $\frac{dy}{dx}$ if $y = e^x \sin x \cos x$. (4)

Q-5 Attempt all questions (14)

- a) Evaluate: $\lim_{n \rightarrow \infty} 4 \left[\frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^4} \right]$ (5)



b) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$. (5)
(i) Find velocity at $t = 0$. (ii) Find acceleration at $t = 1$.

c) Find $\frac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. (4)

Q-6

Attempt all questions

(14)

a) Evaluate: $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ (5)

b) Find $\frac{dy}{dx}$ if $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. (5)

c) Evaluate: $\int \frac{(1-3x)^2}{x^3} dx$ (4)

Q-7

Attempt all questions

(14)

a) Find $\frac{dy}{dx}$ if $y = \log \left[x + \sqrt{x^2 + a^2} \right]$. (5)

b) If $\frac{dy}{dx} = 4x^2 + 6x - 1$ and $y = 5$ when $x = 2$, represent y as a function of x . (5)

c) Evaluate: $\int x e^x dx$ (4)

Q-8

Attempt all questions

(14)

a) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \cos ecx} dx = \frac{\pi}{4}$. (5)

b) Find the area of the circle $x^2 + y^2 = r^2$. (5)

c) Find the equation of a circle passing through point $(-7, 1)$ and centre $(-4, -3)$. (4)

